Journal of Engineering Physics and Thermophysics, Vol. 71, No. 3, 1998

TEMPERATURE FIELD IN A HEAT-TRANSFER WALL WITH A THIN LIQUID FILM EVAPORATING PERIODICALLY FROM ITS SURFACE

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UDC 536.24

Using an approximate method of analysis developed earlier for studying heat-exchange processes occurring with a periodic intensity, the problem of periodic evaporation of a liquid film from the surface of a heat-transfer wall is considered.

In [1] a boundary-value problem is investigated for the equation of heat conduction in a wall with a periodic boundary condition of the third kind. The concept of "true" coefficient of heat transfer is introduced, which is defined as the ratio of the heat-flux density q_δ and the temperature drop ϑ_δ, both taken on the heat transfer surface:

$$\alpha \equiv q_\delta / \vartheta_\delta . \tag{1}$$

Solving the heat-conduction equation, we determine the temperature field in the wall. Knowing it, we can find the averaged coefficient of heat transfer, defined as the quotient of division of the averaged density of the heat flux that passes through the heat-transfer surface ($\langle q_\delta \rangle$) by the averaged "wall–liquid" temperature drop ($\langle \vartheta_\delta \rangle$):

$$\alpha_{\mathrm{m}} = \langle q_\delta \rangle / \langle \vartheta_\delta \rangle . \tag{2}$$

It has been suggested that the quantity α_{m}, which is determined in a traditional heat-exchange experiment and is used in applied calculations, be called the "measured" coefficient of heat transfer.

Due to the initial periodicity of the processes considered, the true heat-transfer coefficient α can always be represented as a superposition of the averaged $\langle \alpha \rangle$ and fluctuating ψ components:

$$\alpha = \langle \alpha \rangle (1 + \psi) . \tag{3}$$

According to Eq. (1), the averaged true coefficient of heat transfer is

$$\langle \alpha \rangle = \langle q_\delta / \vartheta_\delta \rangle . \tag{4}$$

The formal difference between the procedures of averaging Eqs. (2) and (4) tells us that in general the values of α_{m} and $\langle \alpha \rangle$ will not be equal to each other. To evaluate a quantitative measure of their difference, the relative quantity is introduced

$$\varepsilon \equiv \alpha_{\mathrm{m}} / \langle \alpha \rangle . \tag{5}$$

In [1] a proof was obtained in general form for the double inequality that determines the limits of the change in the quantity ε:

$$\langle 1/(1 + \psi) \rangle^{-1} \le \varepsilon \le 1 , \tag{6}$$

From Eq. (6) it follows that the measured value of the heat-transfer coefficient (α_{m}) is smaller than the corresponding averaged true value ($\langle \alpha \rangle$). The quantity ε attains its maximum value ($\varepsilon_{\max} = 1$) in the limiting case

Moscow Power Engineering Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 71, No. 3, pp. 473-476, May-June, 1998. Original article submitted March 14, 1996.

 1062-0125/98/7103-0470$20.00

of infinite thermal conductivity of the wall. Here, we obtain $\alpha_m = \langle\alpha\rangle$. In the alternative case of a wall with zero thermal conmductivity the quantity ε attains its minimum value: $\varepsilon_{min} = [1/(1 + \psi)]^{-1} \leq 1$. For this value there is a maximum difference between α_m and $\langle\alpha\rangle$.

The subject of the analysis in [1] is a boundary-value problem for the equation of heat conduction in a heat-transfer wall that is a plate of thickness δ with boundary condition (1) prescribed on its inner wall (at $x = \delta$).

On the outer surface of the plate (at $x = 0$) one of the standard thermal boundary conditions is prescribed: a) ϑ = const (constant temperature); b) q_0 = const (constant heat-flux density); c) $q_0 = 0$, q_V = const (adiabatic outer surface, constant power of the heat sources). Furthermore, the latter two cases are equivalent as pertains to the field of temperature fluctuations, and therefore only first two thermal boundary conditions should be distinguished: a) ϑ_0 = const; b) q_0 = const.

In [2, 3] a method of approximate solution is suggested for a heat conduction equation with a periodic boundary condition of the third kind. Its efficiency was proved by a comparison with results of a number of exact solutions obtained in [1].

In [4] a model is suggested for a conjugate process of heat exchange occurring with a periodic intensity, namely, the problem of periodic collisions with a heat-transfer wall of a semiinfinite body of liquid with a uniform initial distribution of temperatures. It is assumed that after termination of the interaction the heated body of liquid is instantly replaced by a new-cold one. However, this physically simple formulation of the conjugate problem encounters substantial mathematical difficulties when attempts at a rigorous solution of it are made. Thus, even in a simplified formulation (for a semiinfinite body of the wall) we obtain a system of Wiener–Hopf-type integral equations [5].

Application of the method developed in [2, 3] made it possible to find an approximate analytical solution of the indicated model problem. As a result, in [4] it was found that the relative quantity ε defined by relations (5) and (6) differs very little (by no more than 20%) from unity. This means that the problem of periodic interaction of heat-carrier bodies with a heat-transfer wall, which physically represents "strong" nonstationarity of heat exchange, is characterized by a very "weak" thermal effect of the wall on the averaged heat transfer, i.e., the measured coefficient of heat transfer (α_m) is virtually always equal to the averaged true coefficient ($\langle\alpha\rangle$). We note that here the amplitude of the temperature fluctuations on the heat-transfer surface generally depends noticeably on the thermophysical properties and thickness of the wall.

Below, we consider another possible type of periodic boundary condition of the third kind on a heat-transfer surface, namely, time fluctuations of the true thermal resistance. This approach is of interest, in particular, in the problem of periodic evaporation of a liquid film from a wall surface. We note that the conjugate problem of film evaporation from a wall surface with a uniform initial distribution of temperatures (i.e., in the "single nonperiodic" aspect) was investigated in detail in [6].

The process of evaporation of a thin liquid film is described by the heat-balance relationship

$$r\rho' \frac{dh}{dt} = q, \tag{7}$$

where r is the specific heat of the phase transition and ρ' is the liquid density. From Eq. (7) an equation for the time of complete evaporation of the film t_0 follows:

$$t_0 = \frac{r\rho' h_0}{\langle q\rangle}, \tag{8}$$

where $\langle q\rangle$ is the mean heat-flux density over the time t_0.

It is assumed that after complete evaporation of the film, its place is instantly occupied by a "new film" of the same initial thickness h_0. Here, because of the very small thickness of the film, heat is transferred through it by heat conduction. From this it follows that in the case considered, the magnitude of the averaged (over the time of film evaporation) thermal resistance of the film for any combination of thermophysical properties and thickness of the wall, provided that $q = \text{idem}$, will remain unchanged, $\langle R \rangle \equiv h_0/2\lambda' = \text{idem}$. Thus, the value of R will change periodically in time

$$R = \langle R \rangle (1 + \varphi), \tag{9}$$

where φ is the fluctuating component of the true thermal resistance.

The boundary condition on the heat-transfer surface that reflects the process of periodic evaporation of the thin liquid film is

$$\vartheta_\delta = R q_\delta \tag{10}$$

or in expanded form

$$(1 + \varphi)(1 + \tilde{q}) = \gamma - \tilde{q}^i / \langle \tilde{R} \rangle. \tag{11}$$

Here $\tilde{q} \equiv q'(t)/\langle q \rangle$ is dimensionless fluctuations of the heat-flux density, $\tilde{q}^i$ is the dimensionless temperature fluctuations, $\gamma \equiv R_m/\langle R \rangle$ is the dimensionless coefficient of the thermal effect of the wall, R_m is the experimental thermal resistance of the film, related to the measured coefficient of heat transfer by the relation $\alpha_m R_m \equiv 1$, and

$$\langle \tilde{R} \rangle \equiv \frac{\langle h \rangle}{\lambda'} \left(\frac{\lambda c_p \rho}{t_0} \right)^{1/2}. \tag{12}$$

Use of the approximate method developed in [2, 3] leads to a result similar to the problem of periodic collisions of semiinfinite bodies of liquid with a heat-transfer wall considered in [4], namely that the experimental value of the thermal resistance of the film R_m is virtually equation to the averaged true value $\langle R \rangle$.

Then we will determine the temperature fluctuations on the heat-transfer surface as a function of the parameter $\langle \tilde{R} \rangle$.

Omitting intermediate calculations, we write the final relation that determines the fluctuations of the heat fluxes and the quantity γ:

$$\tilde{q} = -1 + \frac{\gamma + g}{1 + g + \varphi}. \tag{13}$$

The dimensionless parameter is $g \equiv G/\langle \tilde{R} \rangle$, $G = \tanh \delta/\sqrt{at_0}$ at $T_0 = \text{const}$, $G = \text{ctanh}\, \delta/\sqrt{at_0}$ at $q_0 = \text{const}$,

$$\langle \tilde{R} \rangle^2 = \frac{\lambda c_p \rho \langle \vartheta \rangle}{\lambda' \rho' r}, \tag{14}$$

where $\langle \vartheta \rangle$ is the temperature difference averaged over the time of complete evaporation of the film.

For physical applications associated with the process of periodic evaporation of a liquid film from a wall surface under conditions of thermal conjugation [6], it is important to know the law by which the temperature of the heat-transfer surface changes in time. As calculations by relations (13) and (14) show, the following linear approximation is satisfactory:

$$\vartheta \approx \vartheta_{max} - (\vartheta_{max} - \vartheta_{min}) \tilde{t}, \tag{15}$$

where $\tilde{t} \equiv t/t_0$ is the dimensionless time.

The maximum (ϑ_{max}) and minimum (ϑ_{min}) values of the temperature difference are determined from the relations

$$\vartheta_{max} \approx \frac{1 + 2g}{1 + g}; \tag{16}$$

$$\vartheta_{\min} \approx \frac{1}{1+g}. \tag{17}$$

It is seen from Eqs. (16) and (17) that the following versions will be limiting:

1) $g \to 0$ (either the thermal conductivity of the wall tends to infinity, or the wall thickness tends to zero under the thermal boundary condition $T_0 = \text{const}$): $\vartheta_{\max} = \vartheta_{\min} = 1$.

2) $g \to \infty$ (either the thermal conductivity of the wall tends to zero, or the wall thickness tends to zero under the thermal boundary condition $q_0 = \text{const}$): $\vartheta_{\max} \to 2$; $\vartheta_{\min} \to 0$.

The work was carried out with the support from the Russian Fundamental Research Fund, grant No. 95-02-04960-a.

NOTATION

t, time; x, transverse coordinate; q_δ, heat-flux density; ϑ_δ, temperature difference on the heat-transfer surface; α, true coefficient of heat transfer; α_{m}, experimental coefficient of heat transfer; ψ, periodic component of the true coefficient of heat transfer; $\varepsilon = \alpha_{\mathrm{m}}/\langle\alpha\rangle$, ratio of the measured and averaged true values of the heat-transfer coefficient; δ, thickness of the plate; q_V, volumetric power of the heat sources; r, specific heat of the phase change; ρ', density of the liquid; λ', thermal conductivity of the liquid; a', thermal diffusivity of the liquid; t_0, time of complete evaporation of the film; $R = h/\lambda'$, true thermal resistance of the film; φ, periodic component of true thermal resistance of the film; q', fluctuations of the heat-flux density; $\tilde{q} = q'/\langle q\rangle$, dimensionless fluctuations of the heat flux density; ϑ', temperature fluctuations; $\tilde{q}^{\,i} = -\lambda'\vartheta'/(\langle q\rangle\langle\tilde{R}\rangle)$, dimensionless temperature fluctuations; $\gamma \equiv R_{\mathrm{m}}/\langle R\rangle$, dimensionless coefficient of the thermal effect of the wall; R_{m}, measured thermal resistance of the film; $\langle\tilde{R}\rangle$, dimensionless true thermal resistance of the film; $g = G/\langle\tilde{R}\rangle$, dimensionless parameter of the thermal effect of the wall; G, function of the wall thickness effect; $\tilde{t} = t/t_0$, dimensionless time; $\vartheta_{\max}$, $\vartheta_{\min}$, dimensionless values of the maximum and minimum temperature difference in a period, respectively; $\langle\,\rangle$, averaging over the period of the fluctuations. Subscripts: δ, at $x = \delta$; m, measured; max, maximum; min, minimum; 0, initial value.

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